Problem of the Month (September)

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October 2025

The Problem

There are 6 students taking a test consisting of 6 questions. Each correct answer is worth 2 points, and each incorrect answer deducts 1 point. If the total points deducted exceed the total points earned, the score is set to 0.

(1) Show that there are 2 students who have the same score.

(2) Find an example with 5 students who have different scores.

1 Solution to (1)

This is a problem that utilizes the pigeonhole principle. The pigeonhole principle states that if more than n objects are distributed among n boxes, then some box must contain at least two objects. In this problem, our "boxes" are the possible test scores, and the objects we are distributing are students to test scores.

The possibilities for test grades are:
6 correct, 0 wrong $\Rightarrow 6 * 2 + 0 * -1 = 12$
5 correct, 1 wrong $\Rightarrow 5 * 2 + 1 * -1 = 9$
4 correct, 2 wrong $\Rightarrow 4 * 2 + 2 * -1 = 6$
3 correct, 3 wrong $\Rightarrow 3 * 3 + 3 * -1 = 3$
2 correct, 4 wrong $\Rightarrow 2 * 2 + 4 * -1 = 0$
1 correct, 5 wrong $\Rightarrow 1 * 2 + 5 * -1 = -3 \Rightarrow 0$
0 correct, 6 wrong $\Rightarrow 0 * 2 + 6 * -1 = -6 \Rightarrow 0$

As we have a minimum score set to 0, we cannot go lower than that, which is why the last two scores were set to 0. This means our possible test scores are:

12, 9, 6, 3 and 0. This means we have 5 "boxes", and we have 6 students to distrubute to these 5 boxes. As we have more students than boxes, at least two students must share a test score.

To really illustrate this, let's try to fit every student to a different box (I'm going to refer to different test scores for the remainder of this paragraph). Let's label our students, $s_1, s_2, s_3, s_4, s_5, s_6$. Let s_1 be put into any of the 5 boxes, that means there are 4 empty boxes left and 5 students left. Then let's put s_2 into one of the 4 empty boxes left, now we have 4 students left and 3 empty boxes. Putting s_3 into one of the 3 empty boxes, we now have 3 students and 2 empty boxes, then s_4 into an empty box, we then have 2 students left and 1 empty box. Putting s_5 into an empty box, we now have 1 student left, and no empty boxes. Finally, when trying to put s_6 into an empty box, there are no empty boxes left. So we tried to put every student into an empty box, and it was impossible.

2 Solution to (2)

We have 5 students, and let's label them s_1, s_2, s_3, s_4, s_5. Let s_1 have a test score of 12, s_2 have a test score of 9, s_3 have a test score of 6, s_4 have a test score of 3, and s_5 have a test score of 0.

This is possible by letting:
$s_5 \Leftrightarrow$ 6 correct, 0 wrong $\Leftrightarrow$ 12 points
$s_4 \Leftrightarrow$ 5 correct, 1 wrong $\Leftrightarrow$ 9 points
$s_3 \Leftrightarrow$ 4 correct, 2 wrong $\Leftrightarrow$ 6 points
$s_2 \Leftrightarrow$ 3 correct, 3 wrong $\Leftrightarrow$ 3 points
$s_1 \Leftrightarrow$ 2 correct, 4 wrong $\Leftrightarrow$ 0 points