

PROBLEM OF THE MONTH

DEPARTMENT OF MATHEMATICS
ELMHURST UNIVERSITY
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Problem. Let x, y be two positive integers such that $\frac{x+1}{y} + \frac{y+1}{x}$ is an integer. Show that

$$\gcd(x, y) \leq \sqrt{x+y}.$$

Bonus question. Write a program to find all pairs of $1 \leq x, y \leq 10^6$ with the required property. Convince yourself that there are infinitely many x, y with that property.

Scan the following QR code for submission instructions.

