PROBLEM OF THE MONTH

DEPARTMENT OF MATHEMTICS
ELMHURST UNIVERSITY
OCTOBER 2025

Problem. Let $p_1, p_2, \ldots, p_m$ be distinct prime numbers. Show that

$$\log(p_1), \log(p_2), \ldots, \log(p_m) \in \mathbb{R}$$

are linearly independent over $\mathbb{Q}$. (Here, $\log(x)$ denotes the natural logarithm.) Use this fact to show that $\mathbb{R}$ is an infinite dimensional vector space over $\mathbb{Q}$.

Scan the following QR code for submission instructions.

https://qr.link/HZYWnQ

SCAN ME