Coda

How to be a mathematician

It takes a long time to learn to live—by the time you learn your time is gone. I spent most of a lifetime trying to be a mathematician—and what did I learn? What does it take to be one? I think I know the answer: you have to be born right, you must continually strive to become perfect, you must love mathematics more than anything else, you must work at it hard and without stop, and you must never give up.

Born right? Yes. To be a scholar of mathematics you must be born with talent, insight, concentration, taste, luck, drive, and the ability to visualize and guess. For teaching you must in addition understand what kinds of obstacles learners are likely to place before themselves, and you must have sympathy for your audience, dedicated selflessness, verbal ability, clear style, and expository skill. To be able, finally, to pull your weight in the profession with the essential clerical and administrative jobs, you must be responsible, conscientious, careful, and organized—it helps if you also have some qualities of leadership and charisma.

You can't be perfect, but if you don't try, you won't be good enough.

To be a mathematician you must love mathematics more than family, religion, money, comfort, pleasure, glory. I do not mean that you must love it to the exclusion of family, religion, and the rest, and I do not mean that if you do love it, you'll never have any doubts, you'll never be discouraged, you'll never be ready to chuck it all and take up gardening instead. Doubts and discouragements are part of life. Great mathematicians have doubts and get discouraged, but usually they can't stop doing mathematics anyway, and, when they do, they miss it very deeply.

"Mathematician" is, to be sure, an undefined term, and it is possible that some people called mathematicians nowadays (or ever) do not (or did not) love mathematics all that much. A spouse unsympathetic to mathematics demands equal time, a guilty parental conscience causes you to play catch with your boy Saturday afternoon instead of beating your head against the brick wall of that elusive problem--family, and religion, and money, comfort, pleasure, glory, and other calls of life, deep or trivial, exist for all of us to varying degrees, and I am not saying that mathematicians always ignore all of them. I am not saying that the love of mathematics is more important than the love of other things. What I am saying is that to the extent that one's loves can be ordered, the greatest love of a mathematician (the way I would like to use the term) is mathematics. I have known many mathematicians, great and small, and I feel sure that what I am saying is true about them. To mention some famous names: I'd be very surprised if Marston Morse, and André Weil, and Hermann Weyl, and Oscar Zariski didn't agree with me.

Mind you, I am not recommending or insisting that you love mathematics. I am not issuing an order: "If you want to be a mathematician, start loving mathematics forthwith"—that would be absurd. What I am saying is that the love of mathematics is a hypothesis without which the conclusion doesn't follow. If you want to be a mathematician, look into your soul and ask yourself how much you want to be one. If the wish isn't very deep and very great, if it is not, in fact, maximal, if you have another desire that takes precedence, or even more than one, then you should not try to be a mathematician. The "should" is not a moral one; it is a pragmatic one. I think that you would probably not succeed in your attempt, and, in any event, you would probably feel frustrated and unhappy.

As for working hard, I got my first hint of what that means when Carmichael told me how long it took him to prepare a fifty-minute invited address. Fifty hours, he said; an hour of work for each minute of the final presentation. When, many years later, six of us wrote our "history" paper ("American mathematics from 1940..."), I calculated that my share of the work took about 150 hours; I shudder to think how many man-hours the whole group put in. A few of my hours went toward preparing the lecture (as opposed to the paper). I talked it, the whole thing, out loud, and then, I talked it again, the whole thing, into a dictaphone. Then I listened to it, from beginning to end, six times—three times for spots that needed polishing (and which I polished before the next time), and three more times to get the timing right (and, in particular, to get a feel for the timing of each part). Once all that was behind me, and I had prepared the transparencies, I talked the whole thing through one final rehearsal time (by myself—no audience). That's work.

Archimedes taught us that a small quantity added to itself often enough becomes a large quantity (or, in proverbial terms, that every

little bit helps). When it comes to accomplishing the bulk of the world's work, and, in particular, the work of a mathematician, whether it is proving a theorem, writing a book, teaching a course, chairing a department, or editing a journal, I claim credit for the formulation of the converse: Archimedes's way is the only way to get something done. Do a small bit, steadily every day, with no exception, with no holiday. As an example, I mention the first edition of my *Hilbert Space Problem Book*, which had 199 problems. I wrote most of the first draft during my Miami year, and I forced myself, compulsively, to write a problem a day. That doesn't mean that it took 199 days to write the whole book —the total came to about three times that many.

As for "never give up", that doesn't need explanation, and I have been trying to illustrate it with anecdotal evidence all along, but here, just for fun, is a pertinent little story. Somewhere around 1980 I was invited to give a talk to a "general" audience, and, having given it, I wrote it up and submitted it for publication in The Mathematics Teacher. In due course I received reports from a couple of referees; they read, in part, as follows. "The author apparently feels she/he is illustrating the thrill and power of abstraction. While his/her examples contain the potential for doing that, I don't feel that the presentation creates that effect.... Much of the problem with the paper is that of a meandering style. The progression of ideas is not clear.... The mathematical topics focused on are of only moderate interest." The paper was firmly rejected. I didn't give up—I just shrugged my shoulders, and submitted the same article, word for word, to what was then called The Two-Year College Mathematics Journal. It was accepted and printed, and, a year after that, it received the Pólya award from the Association.

All these prescriptions and descriptions about how to be a mathematician arose, inevitably, from my own attempts to become one. Nobody can tell you what mathematicians should do, and I am not completely sure I know what in fact they do—all I can really say is what I did.

How close did I come? What is my total mathematical contribution? The first answers that occur to me are a small but pretty proof (the monotone class theorem), a few reasonable theorems (mainly in my ergodic papers "Approximation theories..." and "In general a measure-preserving theorem is mixing"), and a good idea in logic (polyadic algebras).

One thing that I am pretty good at is asking questions. Given a mathematical problem that I understand the statement of, one whose history I know something about and that I spent some time studying, one for which I am moderately up to date with the standard techniques —given those circumstances, I have some talent for identifying and formulating the central questions. If I look hard at a question for a month, try to answer it, and fail, then I am confident that it is not trivial. I am confident that a question like that is fit for mathematicians much better than I; when one of them solves it, he is bound to feel at least a small tingle of pride and pleasure. (Examples: the power inequality, and the Weyl–von Neumann theorem for normal operators.)

Related to the questions I have asked are the concepts I discovered and introduced, notably subnormal and quasitriangular operators, and, possibly, capacity in Banach algebras. Important theories have grown out of such concepts; I think it's fair to call them contributions.

I wrote a few good surveys and some pretty good books. Perhaps the best are *Finite-Dimensional Vector Spaces* and *A Hilbert Space Problem Book*—but then my vote on such matters is probably the one that weighs the least.

My most nearly immortal contributions are an abbreviation and a typographical symbol. I invented "iff", for "if and only if"—but I could never believe that I was really its first inventor. I am quite prepared to believe that it existed before me, but I don't *know* that it did, and my invention (re-invention?) of it is what spread it through the mathematical world. The symbol is definitely not my invention—it appeared in popular magazines (not mathematical ones) before I adopted it, but, once again, I seem to have introduced it into mathematics. It is the symbol that sometimes looks like □, and is used to indicate an end, usually the end of a proof. It is most frequently called the "tombstone", but at least one generous author referred to it as the "halmos".

That's it; that's a life, that's a career. I was, in I think decreasing order of quality, a writer, an editor, a teacher, and a research mathematician.

What's next? The writing of this book took a lot out of me. It took a year and a half of time and energy, during which I paid no attention to research. It was a deliberate gamble. I wanted to write this book, I wasn't at all sure that I could do it the way I dreamt, I wasn't sure I could tell the audience I had in mind what I wanted to say. If it turns out that I succeeded, I'll be happy; if not, I'll be sad. But, in either case, I'm not ready to crawl into a hole and pull it in after me. I'd like to write some more mathematics, to teach some more, and, who knows, even to prove a theorem. I'm going to try, that's for sure. I thought, I taught, I wrote, and I talked mathematics for fifty years, and I am glad I did. I wanted to be a mathematician. I still do.